TK Hemmick, S Taneja, A Deshpande, P. Kline, B. Chonigman

Chiapas

Hank Levy…1927 - 2001

- Hank Levy: American Jazz Composer; leading author of jazz in "Time" (odd time signatures)
- An incident occurred when Stan Kenton's Band first recorded the Levy chart "*Chiapas*":
  - The lead sax player was unable to play the music and stormed off. Hours later he came back having transcribed the music to 4/4 time.
- Chiapas for EIC converts kinematics and more importantly RESOLUTIONS from the physical variables (x,Q^2) to (p,θ)

Hemmick played alto & bari sax in Hank Levy's College Jazz Band (band 2 of 3) in 1980

Goals

- The EIC "Golden Measurement" for determining tracking requirements is $F_L(x,Q^2)$
- This measurement requires that we measure the reduced cross section $\sigma_{red}(x,Q^2)$ at various beam kinematics so as to find the variation over a range in inelasticity (y) and thereby measure F_L
- One can semi-analytically factorize the error in and reduced cross section measurement due to experimental measures.

Some math....

- $\sigma_{red} \equiv \frac{d^2\sigma}{dxdQ^2}\left(\frac{d^2\sigma_{Mott}}{dxdQ^2}\right)^{-1} = \frac{Q^4 x}{2\pi\alpha^2 Y_+}\frac{d^2\sigma}{dxdQ^2}$
- $\sigma_{red} = F_2(x,Q^2) - \frac{y^2}{Y_+}F_L(x,Q^2)$
- The measurement is made by counting (dN) in bins of some width $\Delta\ln(x)$ by $\Delta\ln(Q^2)$ (squares on log-log)
- $d^2N = \mathcal{L}\frac{d^2\sigma}{dxdQ^2}dxdQ^2 =$

  $\mathcal{L}\left(F_2(x,Q^2) - \frac{y^2}{Y_+}F_L(x,Q^2)\right)\frac{2\pi\alpha^2 Y_+}{Q^4 x}dxdQ^2 =$

  $\mathcal{L}\left(F_2(x,Q^2) - \frac{y^2}{Y_+}F_L(x,Q^2)\right)\frac{2\pi\alpha^2 Y_+}{Q^2}d\ln(x)d\ln(Q^2)$
- $\frac{d^2N}{d\ln(x)d\ln(Q^2)} = \mathcal{L}\left(F_2(x,Q^2) - \frac{y^2}{Y_+}F_L(x,Q^2)\right)\frac{2\pi\alpha^2 Y_+}{Q^2}$

Parameterized: e.g. MRST2002 (NLO)

Simple Kinematics

Errors due to stats & resolution:

- $\frac{d^2N}{dln(x)dln(Q^2)} = \mathcal{L}\left(F_2(x,Q^2) - \frac{y^2}{Y_+}F_L(x,Q^2)\right)\frac{2\pi\alpha^2 Y_+}{Q^2} \equiv \mathcal{L}M(x,Q^2) \equiv \mathcal{L}\bar{M}(p,\theta)$
- Error Summary:
  - $\frac{\delta\left(\frac{d^2N}{dln(x)dln(Q^2)}\right)}{\frac{d^2N}{dln(x)dln(Q^2)}} = \frac{\frac{\partial\bar{M}}{\partial p}\delta p}{\bar{M}} \oplus \frac{\frac{\partial\bar{M}}{\partial\theta}\delta\theta}{\bar{M}} \oplus \frac{1}{\sqrt{\mathcal{L}\bar{M}(p,\theta)\Delta\ln(x)\Delta\ln(Q^2)}}$
- Fractional error due ONLY to momentum:
  - $\frac{\frac{\partial\bar{M}}{\partial p}\delta p}{\bar{M}} = \frac{\partial\ln(\bar{M})}{\partial p}\delta p$
- Fractional error due ONLY to direction:
  - $\frac{\frac{\partial\bar{M}}{\partial\theta}\delta\theta}{\bar{M}} = \frac{\partial\ln(\bar{M})}{\partial\theta}\delta\theta$

Computing Error Targets:

- If we assume that any of these terms should be set to some constant fractional error ε, we can then solve of the δp and δθ requirement.
- $\delta p = \varepsilon \left(\frac{\partial \ln(\bar{M})}{\partial p} \right)^{-1}; \frac{\delta p}{p} = \varepsilon \frac{1}{p} \left(\frac{\partial \ln(\bar{M})}{\partial p} \right)^{-1}$
- $\delta \theta = \varepsilon \left(\frac{\partial \ln(\bar{M})}{\partial \theta} \right)^{-1}$

Goals of Chiapas

- Calculate, as a function of (p,θ):
  - $\frac{1}{p}\left(\frac{\partial \ln(\bar{M})}{\partial p}\right)^{-1}$ & $\left(\frac{\partial \ln(\bar{M})}{\partial \theta}\right)^{-1}$ & $\frac{1}{\sqrt{\bar{M}(p,\theta)}}$
- Provide user code to:
  - Accept a target value of epsilon.
  - Plot target curves of $\frac{\delta p}{p}$ & $\delta\theta$ as functions of momentum for bins in θ.
  - Overlay statistical error profiles on the prior plots with user-supplied values of $\mathcal{L}$, Δln(x), and Δln(Q^2).
- User must select ε intelligently.

NOTE: $(\hbar c)^2 = 3.89\ x\ 10^{11}\text{fb GeV}^2$

Considerations for ε

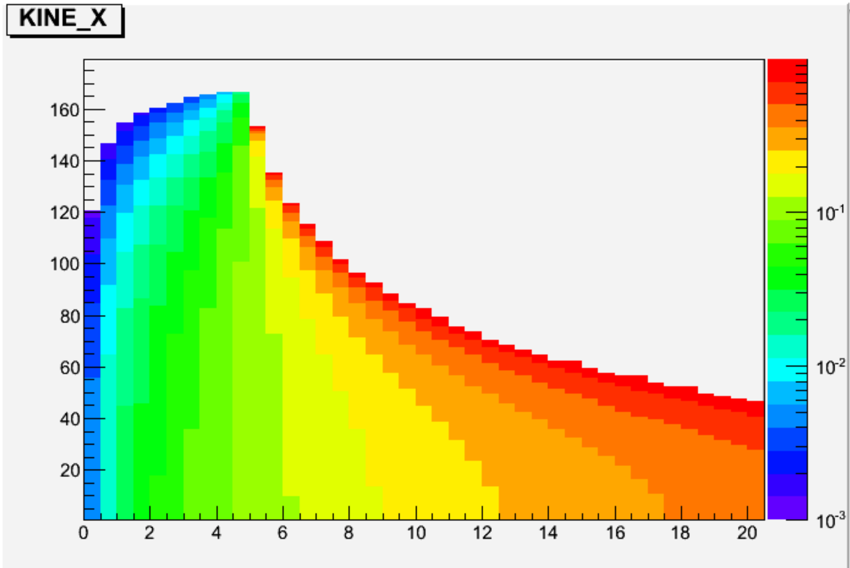
- In a real experiment, when you know your finite resolution you can correct for it!
- Thus, one might say that a desired result at 1% comes from a spectrometer with $\varepsilon=0.05$
- The knowledge of physicists who have done this before is required to establish e.
- The result of the algebra (barring bugs) should be correct.

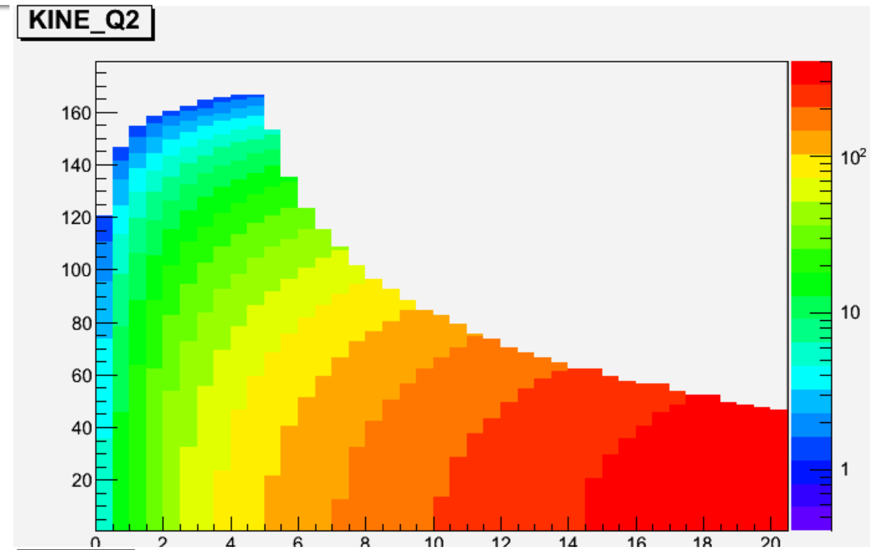
We shall use $\varepsilon=0.05$ in the calculations here…assuming that correction makes 1% result.

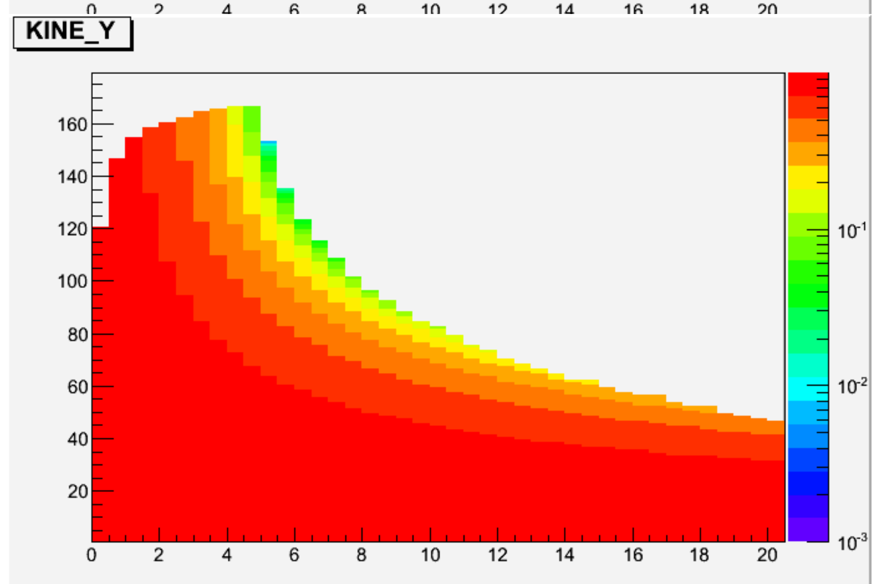
Calculation Sets:

- Early eRHIC running will start @ 5 GeV and increase to ~10 GeV via operations funds
  - E_{proton} = 50, 100, & 250 GeV
  - $\mathcal{L}$ = 10 fb^{-1} different calendar times for each E_{proton}
  - 10 bins per decade in x and Q^2
    - $\Delta\ln(x) = \Delta\ln(Q^2) = 0.23$
- These are all reasonable estimates.
- We'll use ε=0.05 targeting an after-correction precision of 1%.

Kinematical Guidelines 5x50

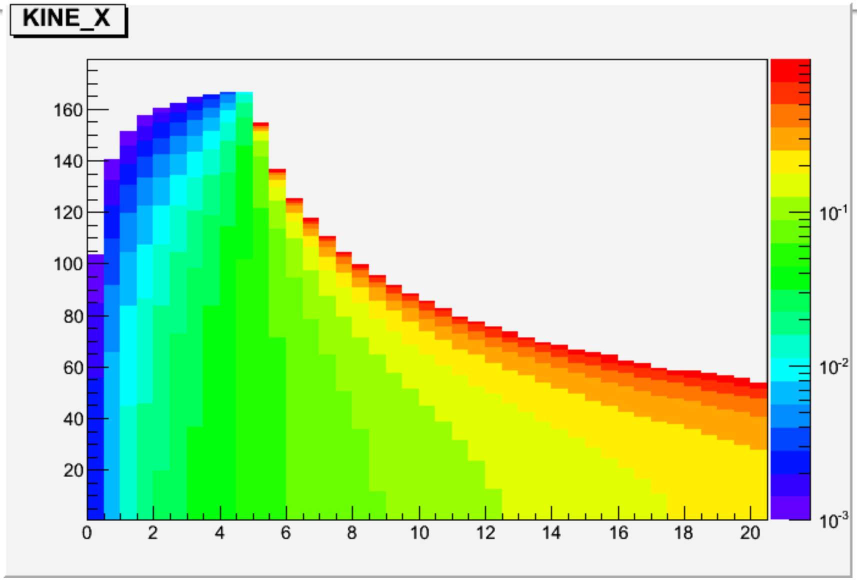


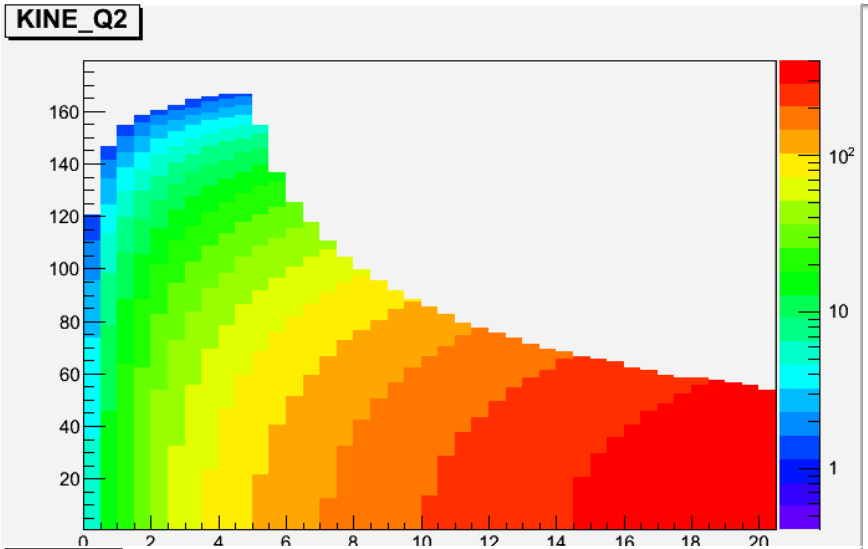


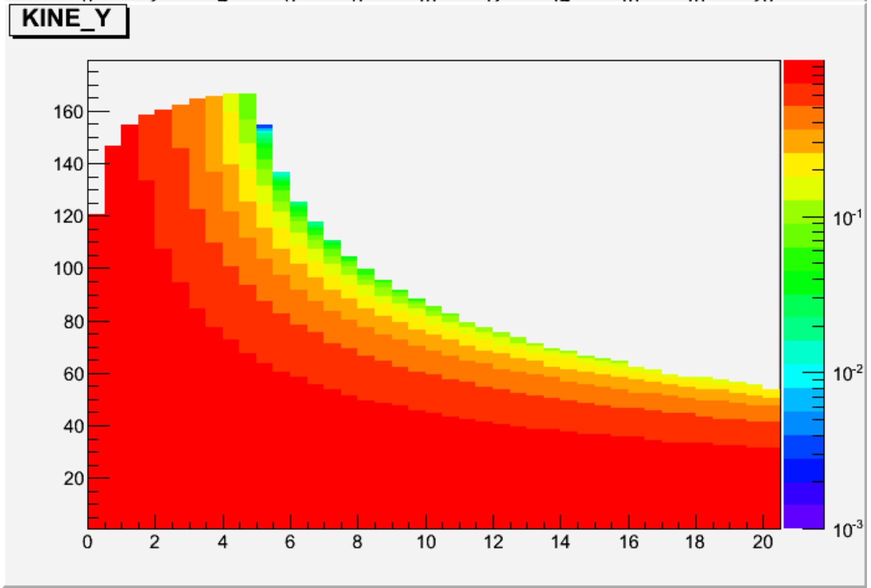


- Plots show which kinematics are measured by electrons @ each (θ, p_T)

Kinematical Guidelines 5x100







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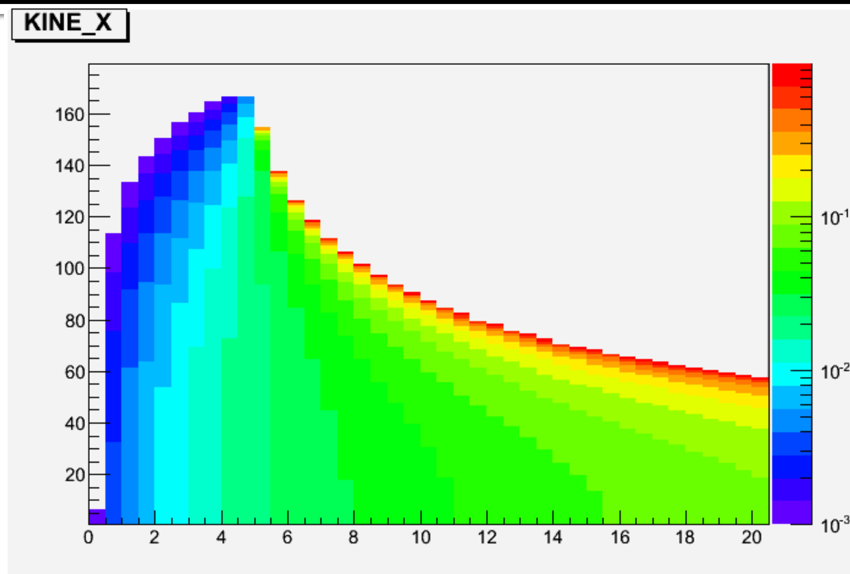
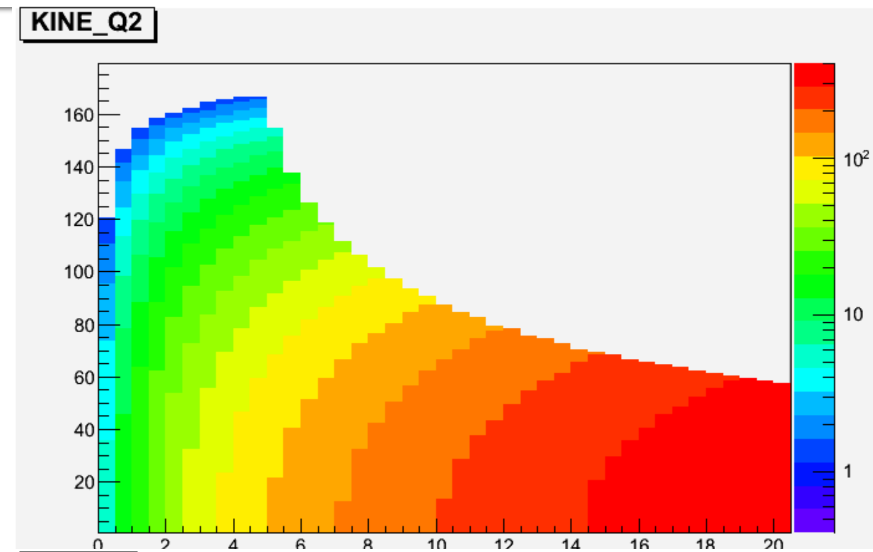
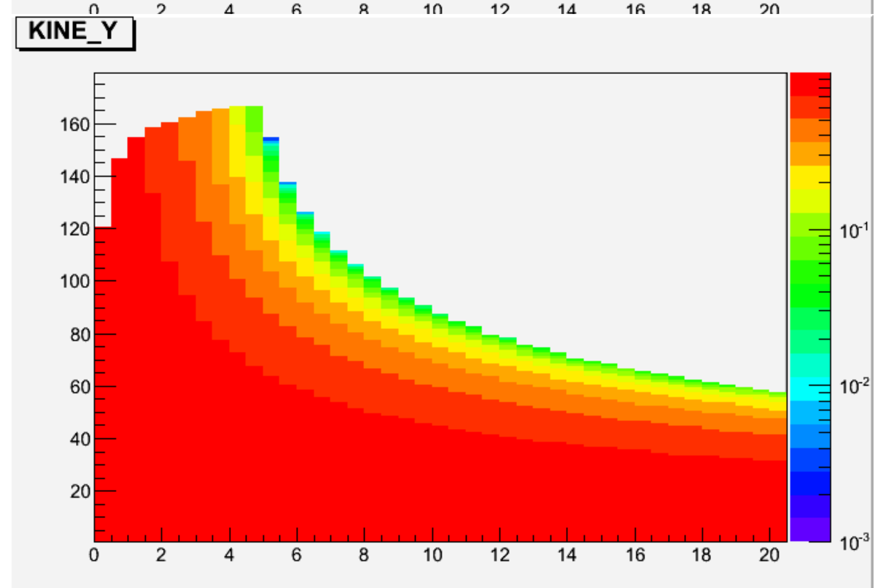
Kinematical Guidelines 5x250

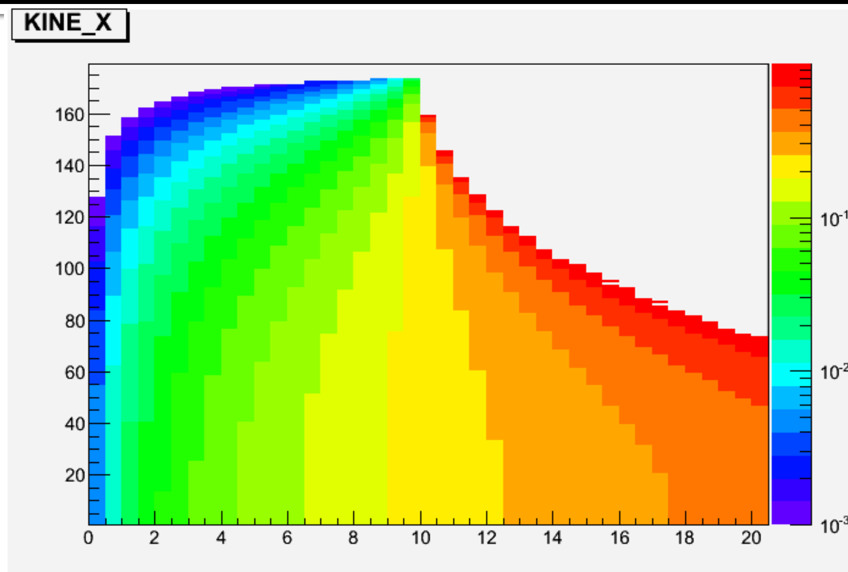


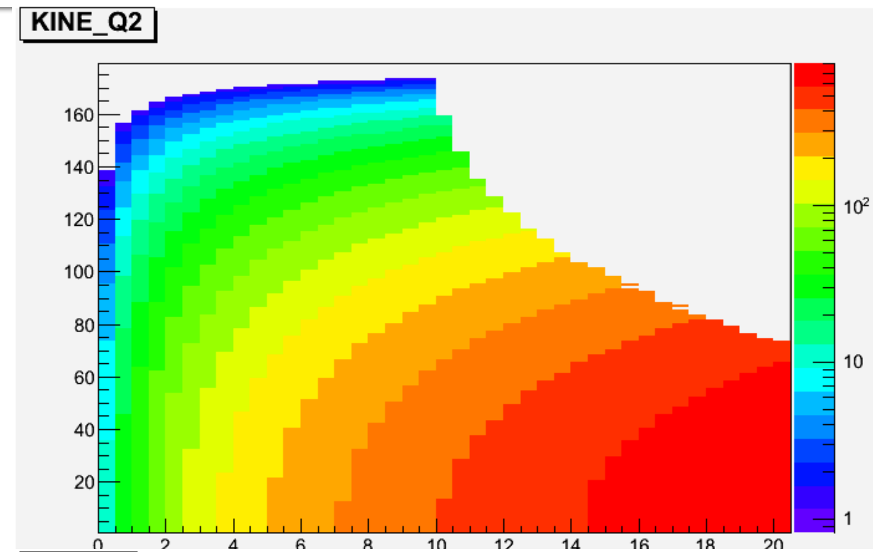


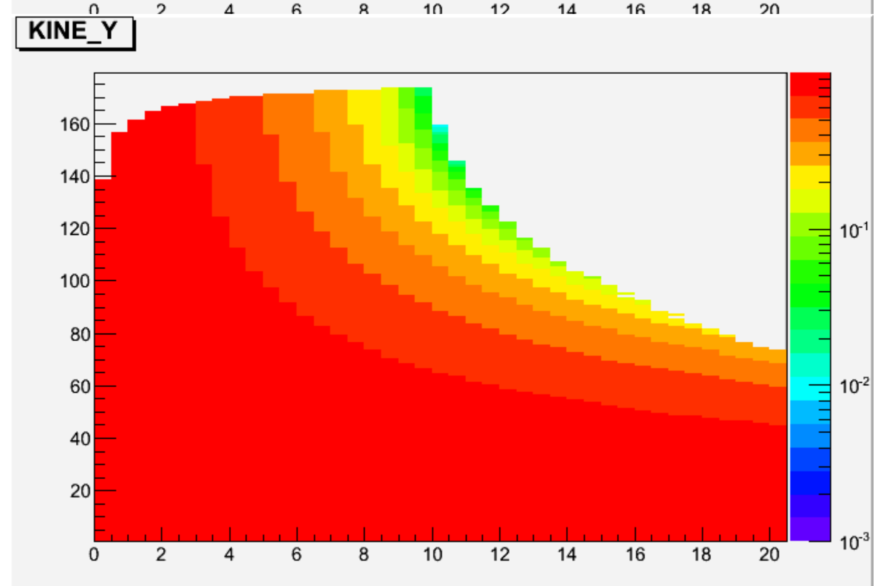


- Plots show which kinematics are measured by electrons @ each (θ, p_T)

Kinematical Guidelines 10x50

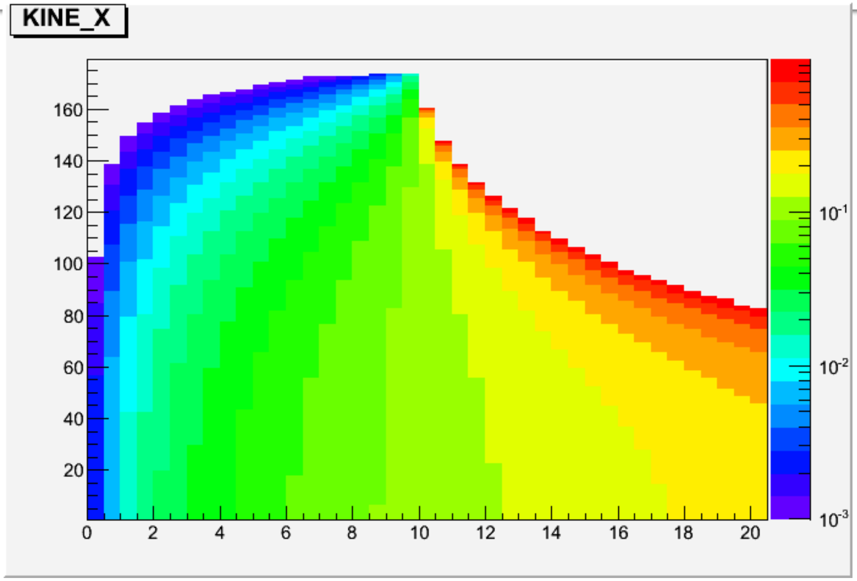


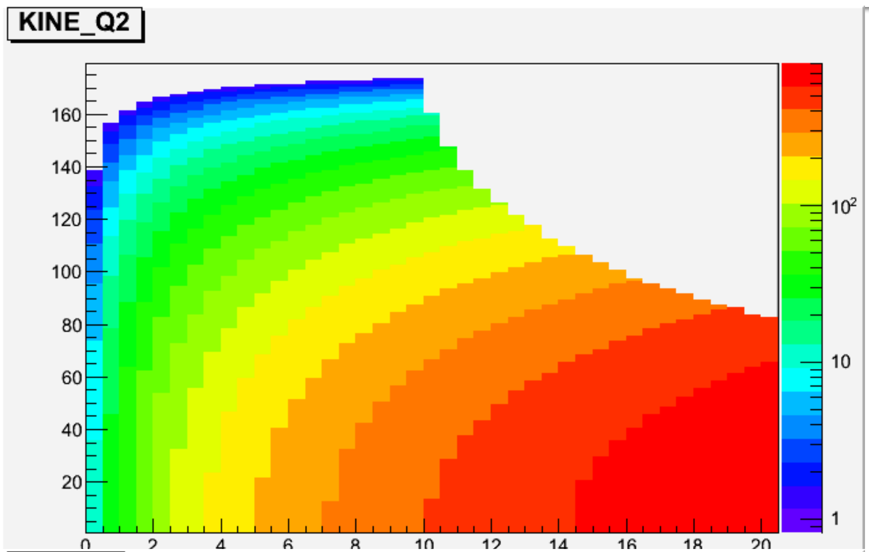


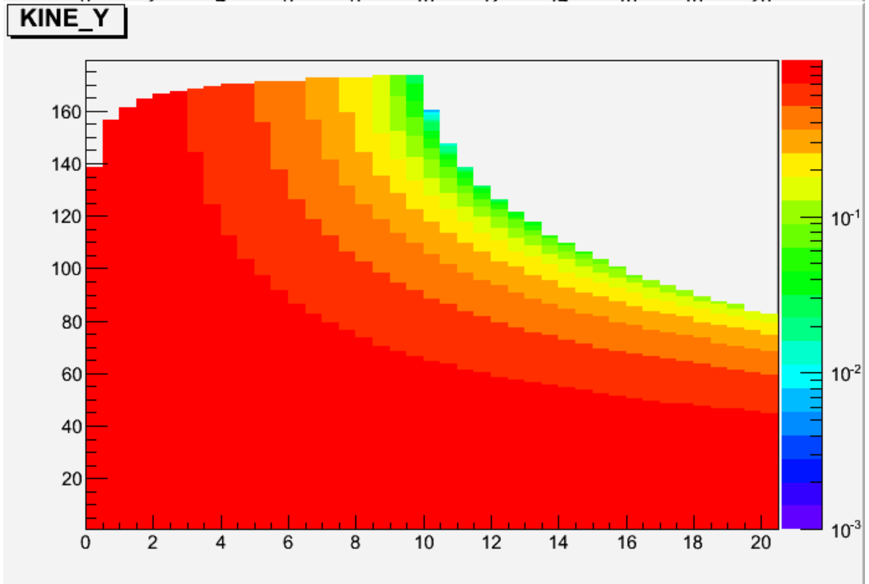


- Plots show which kinematics are measured by electrons @ each (θ,p_T)

Kinematical Guidelines 10x100

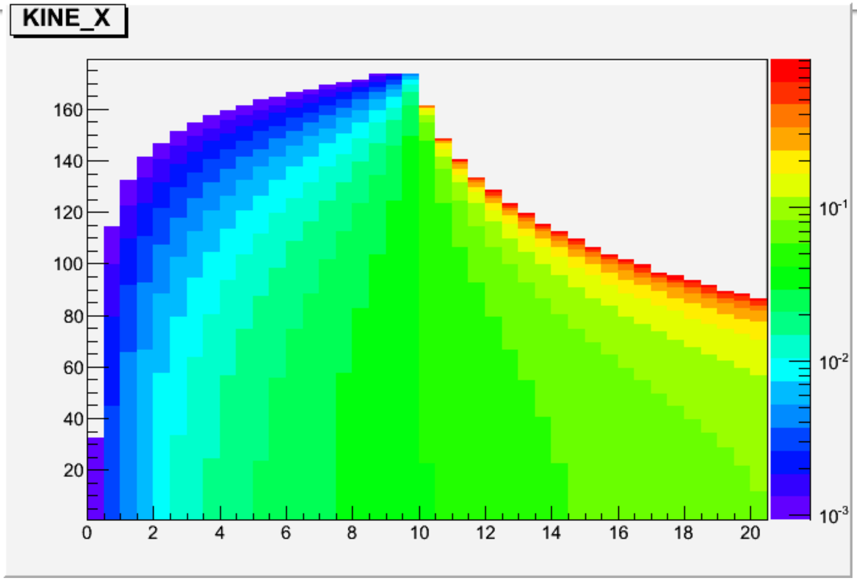


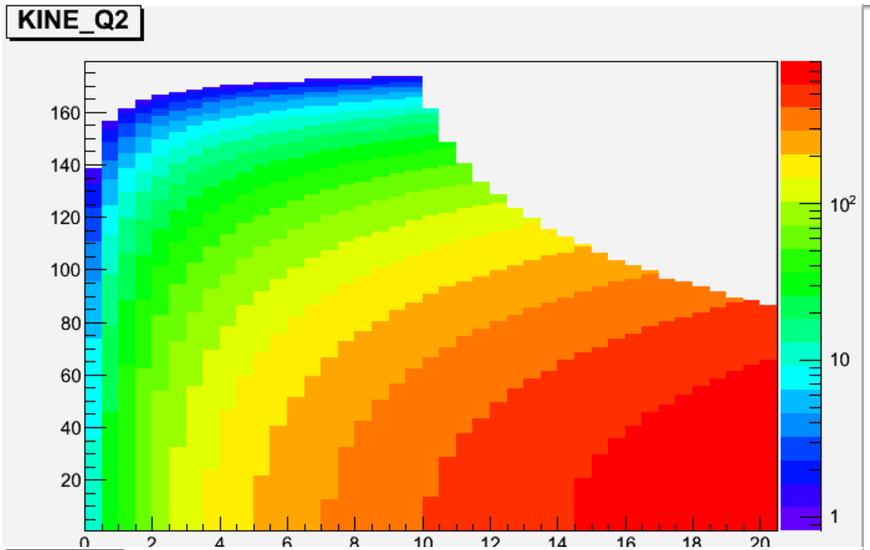


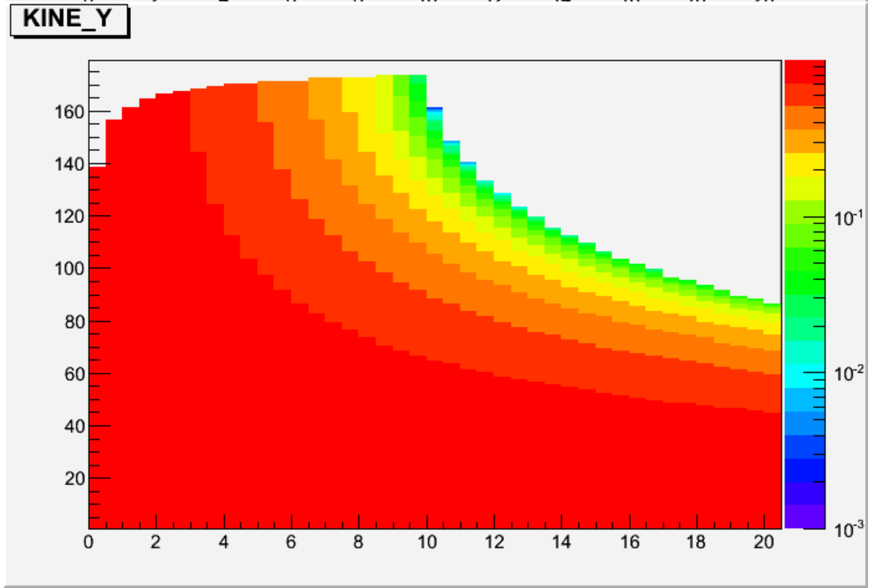


- Plots show which kinematics are measured by electrons @ each (θ,p_T)

Kinematical Guidelines 10x50

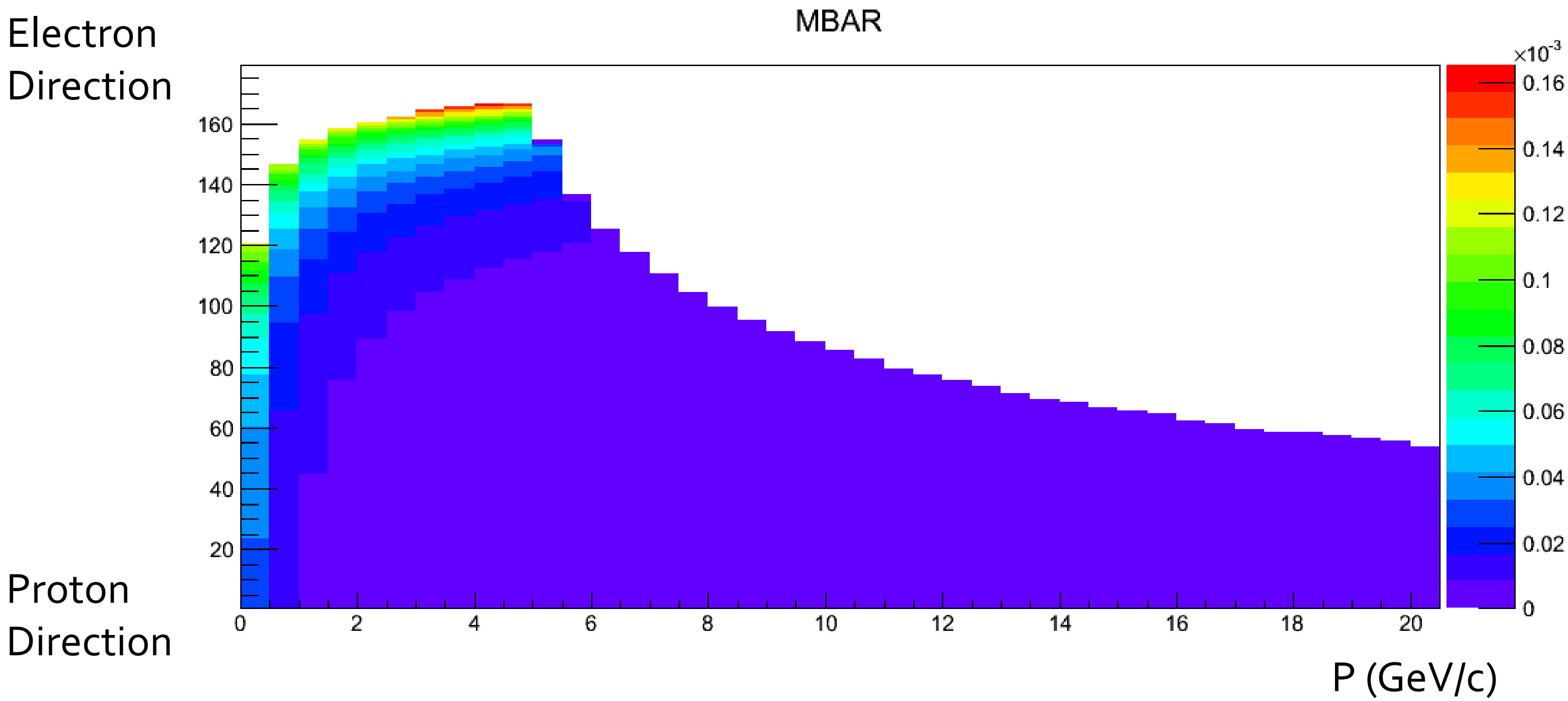






- Plots show which kinematics are measured by electrons @ each (θ,p_T)

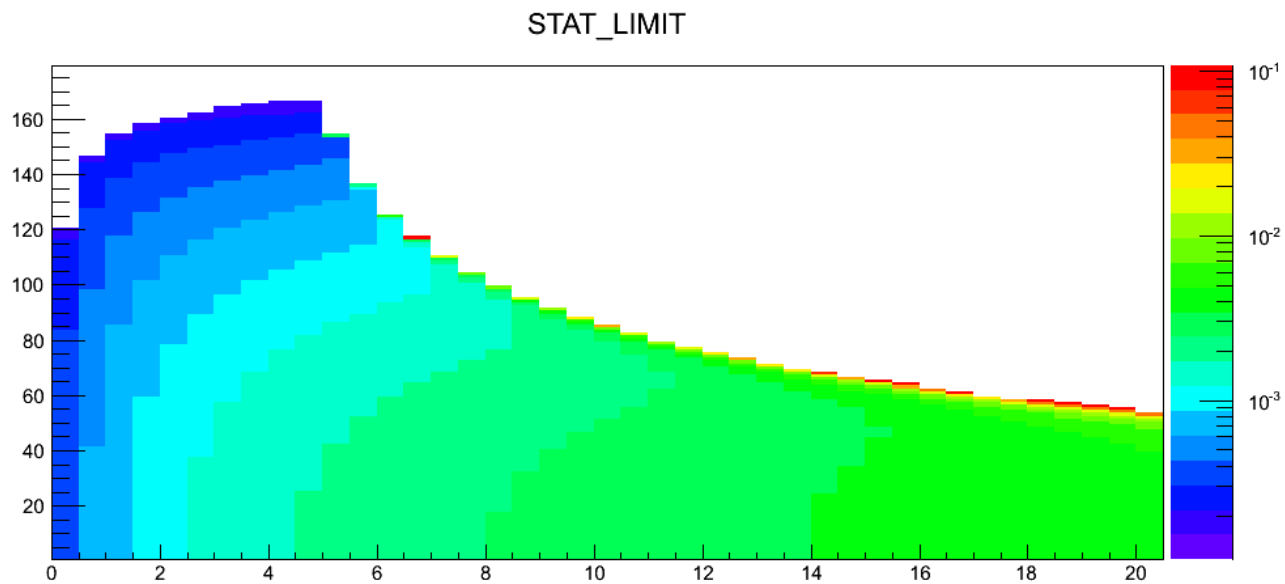
MBAR(p,θ) 5x100 GeV



- The kinematical edges are based upon where MRST throws complaints.

Statistical Limits

Electron Direction



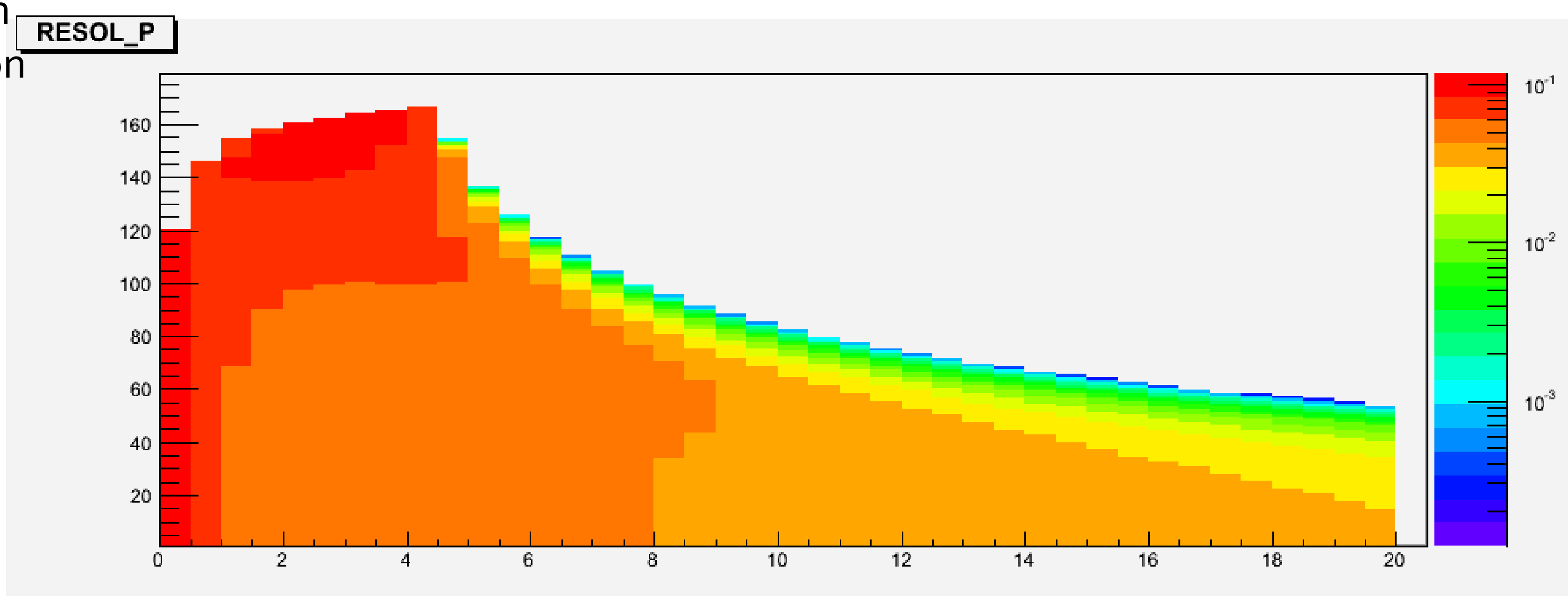
Proton Direction

P (GeV/c)

- The Z-scale is the fractional error from 10 fb^{-1} measured into bins of 10 per decade of x, Q^2

Momentum Resolution

Electron Direction

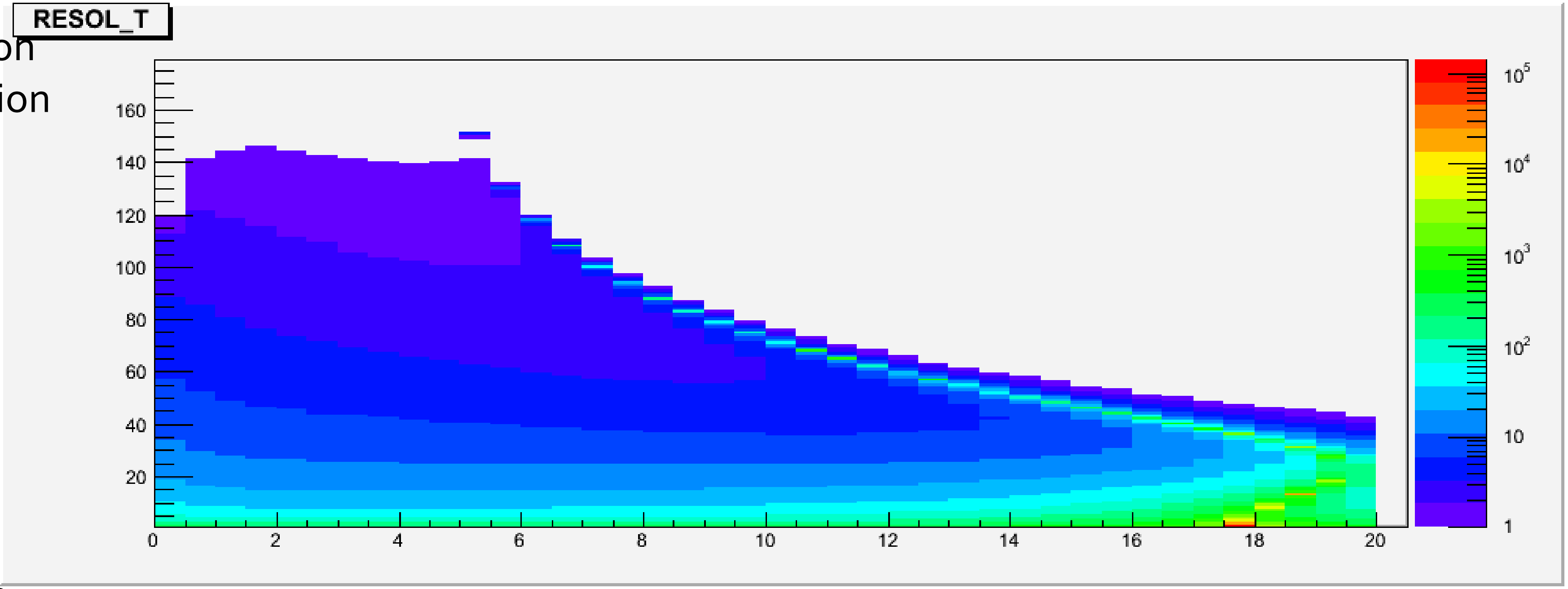


Proton Direction

- The color scale is δp/p limit required to produce e=0.01 performance for 5x100.
- Structure near edges could be mistakes in derivative at kinematical boundary…should be checked.

Angular Resolution

Electron Direction

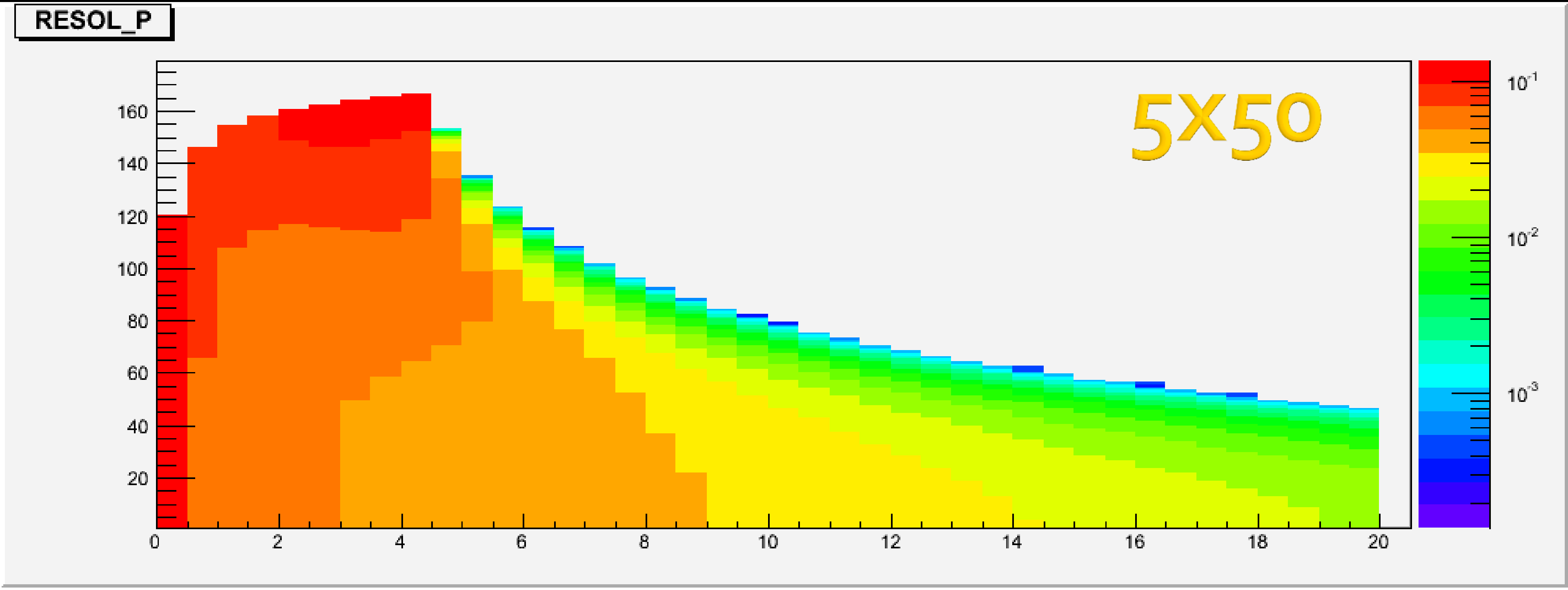


Proton Direction

- This is the angular resolution in degrees (sorry) across the spectrometer.
- Again, slightly strange at kinematical boundary.

Remarks

- Stats easily beat 1%.
- Therefore spectrometer resolution is the key factor in determining precision.
- Summarize resolutions on next 6 slides.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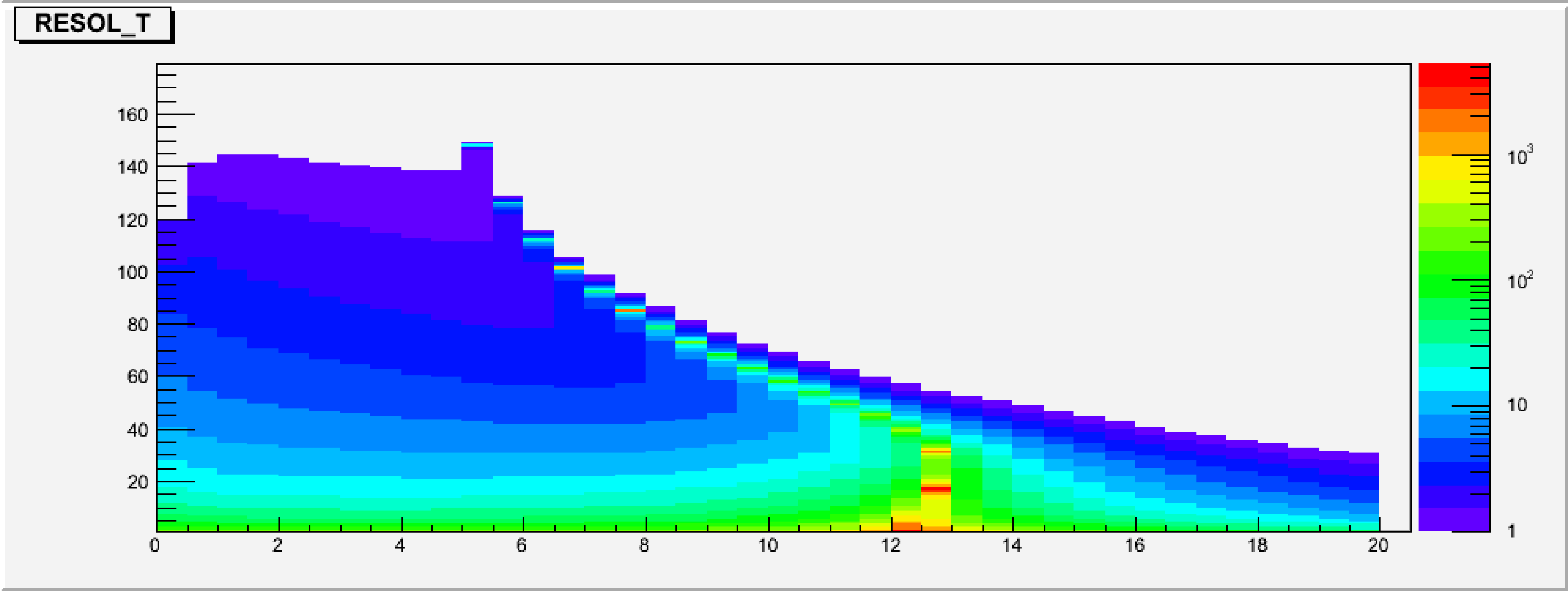
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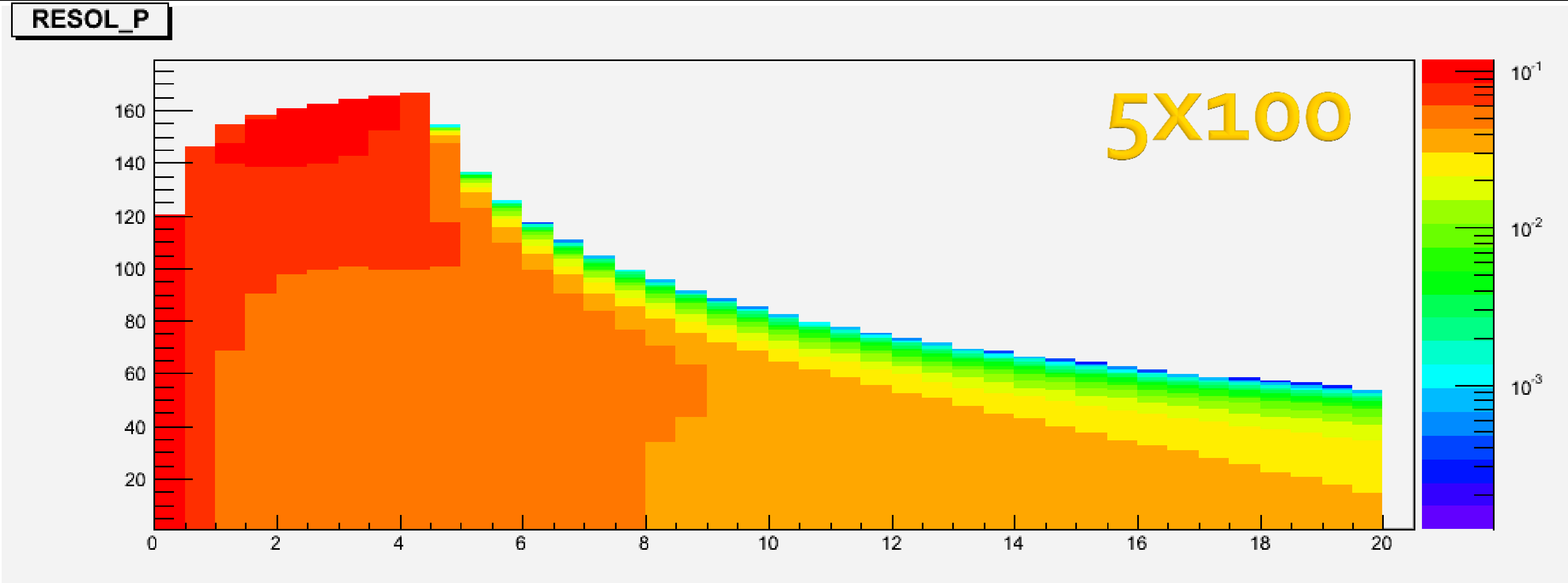
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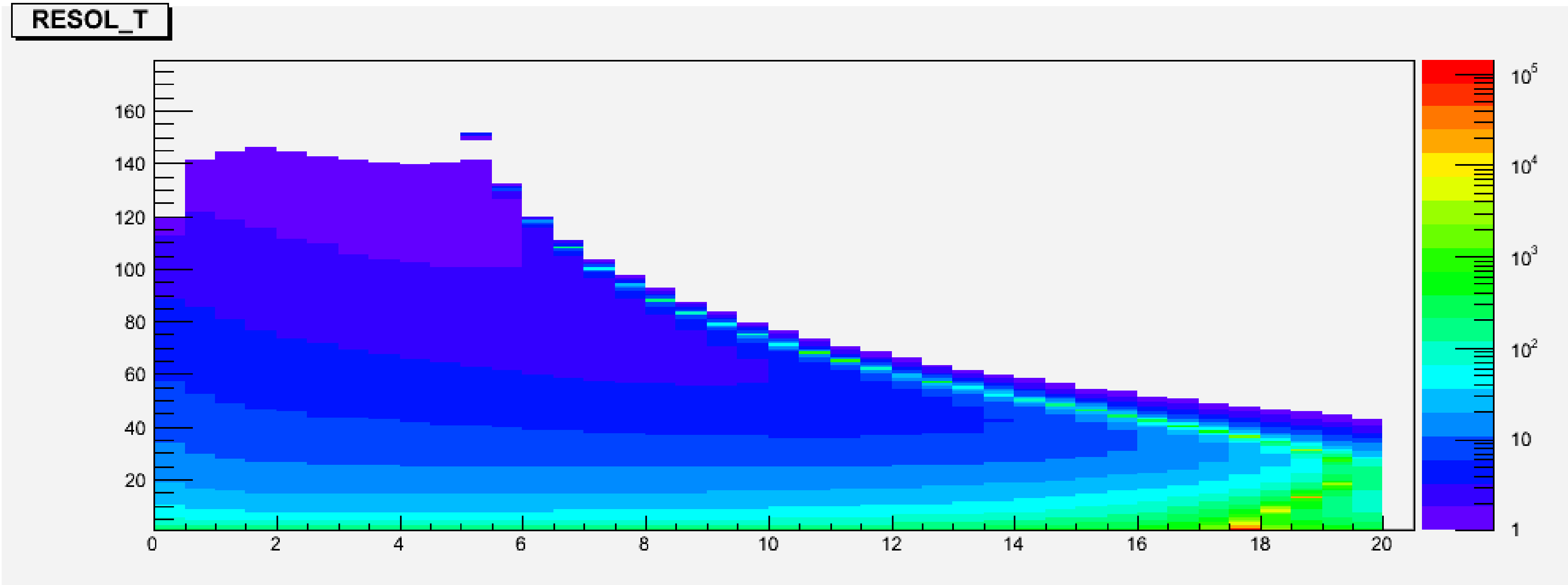
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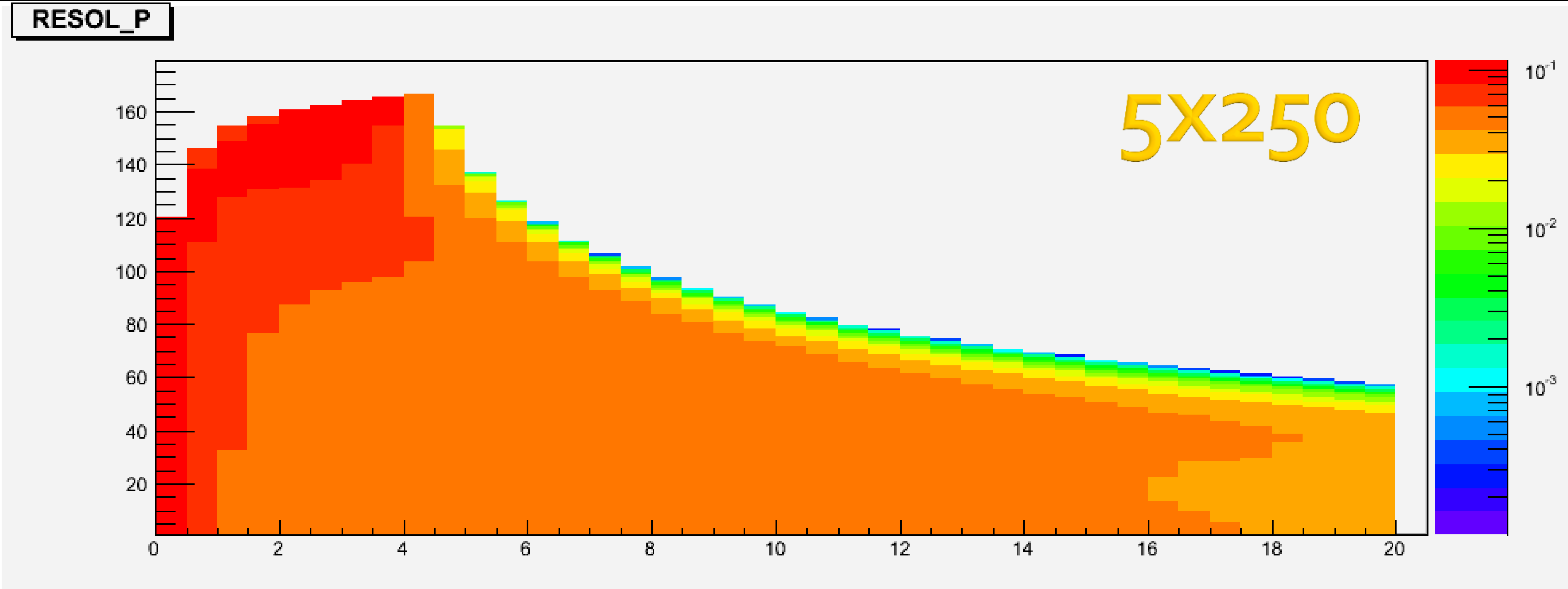
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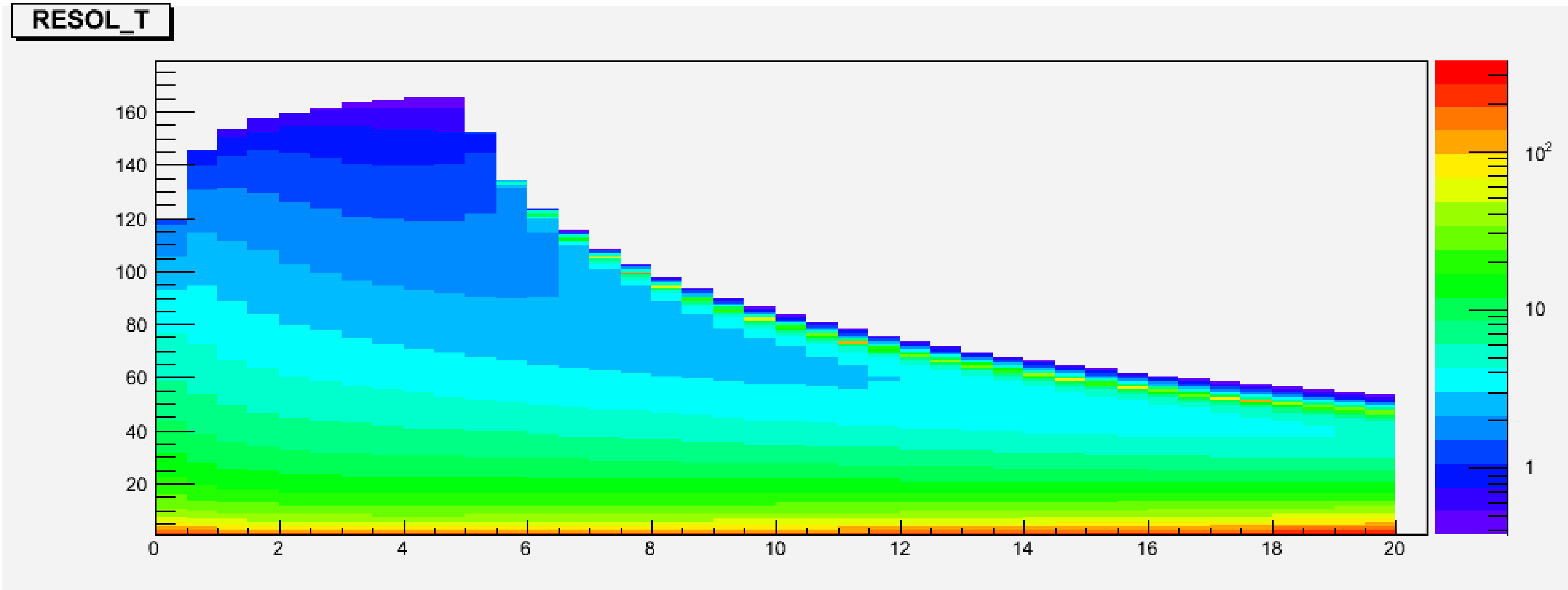
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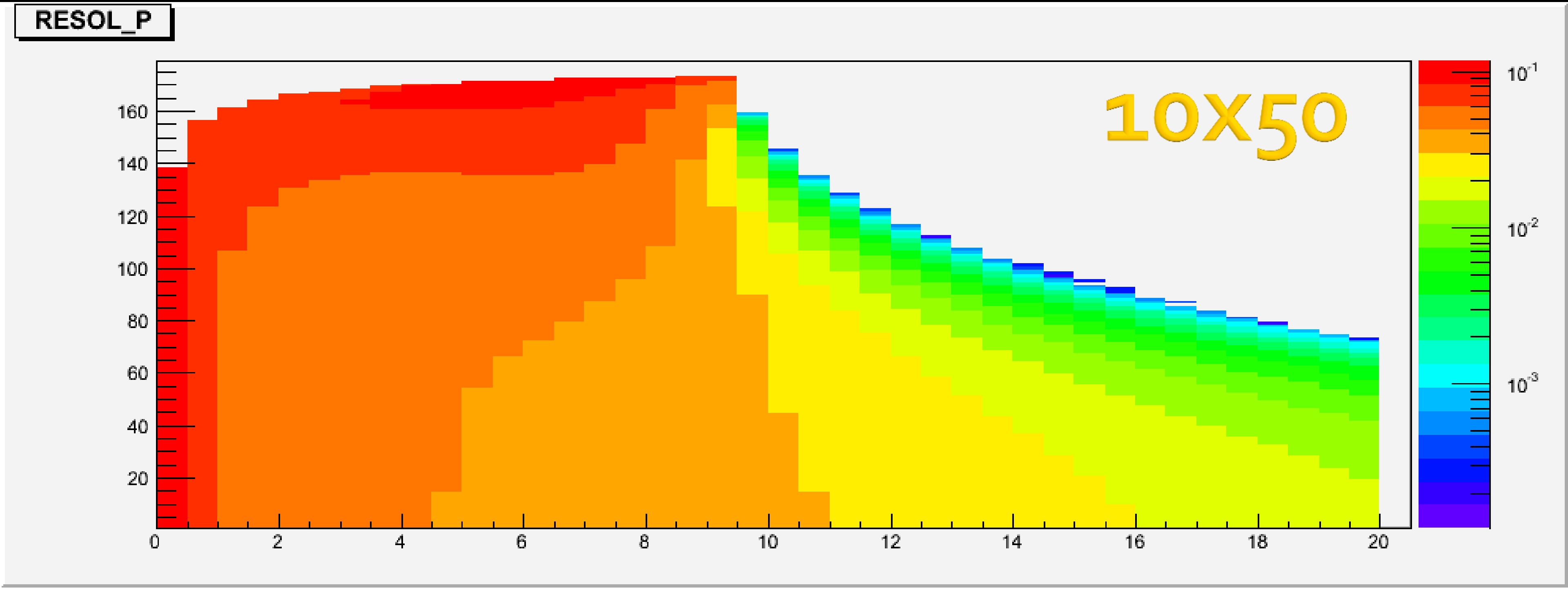

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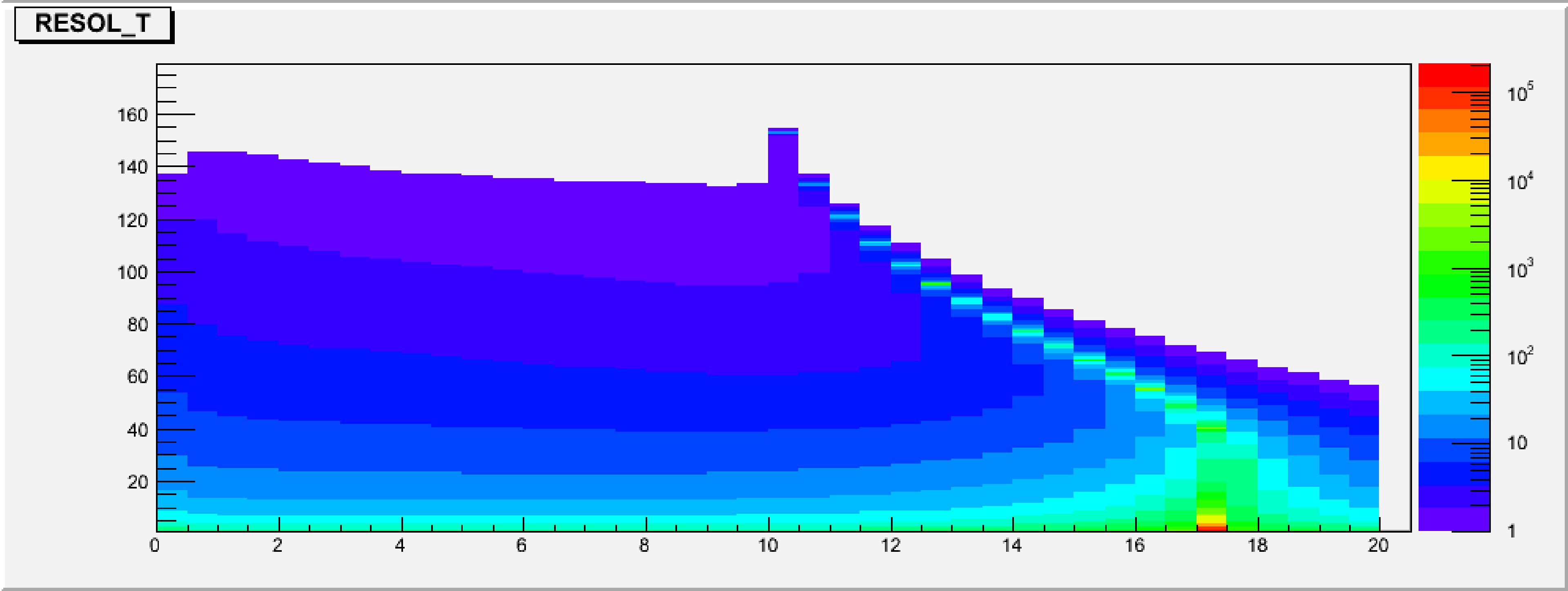

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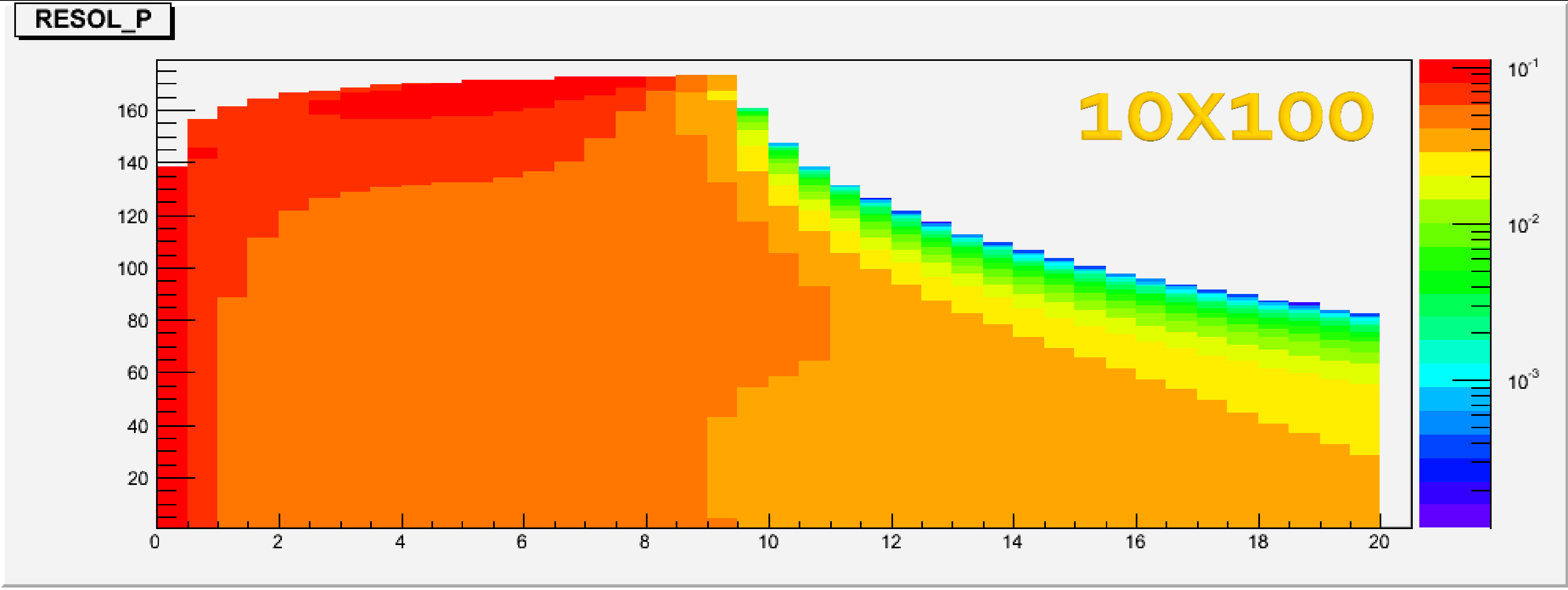
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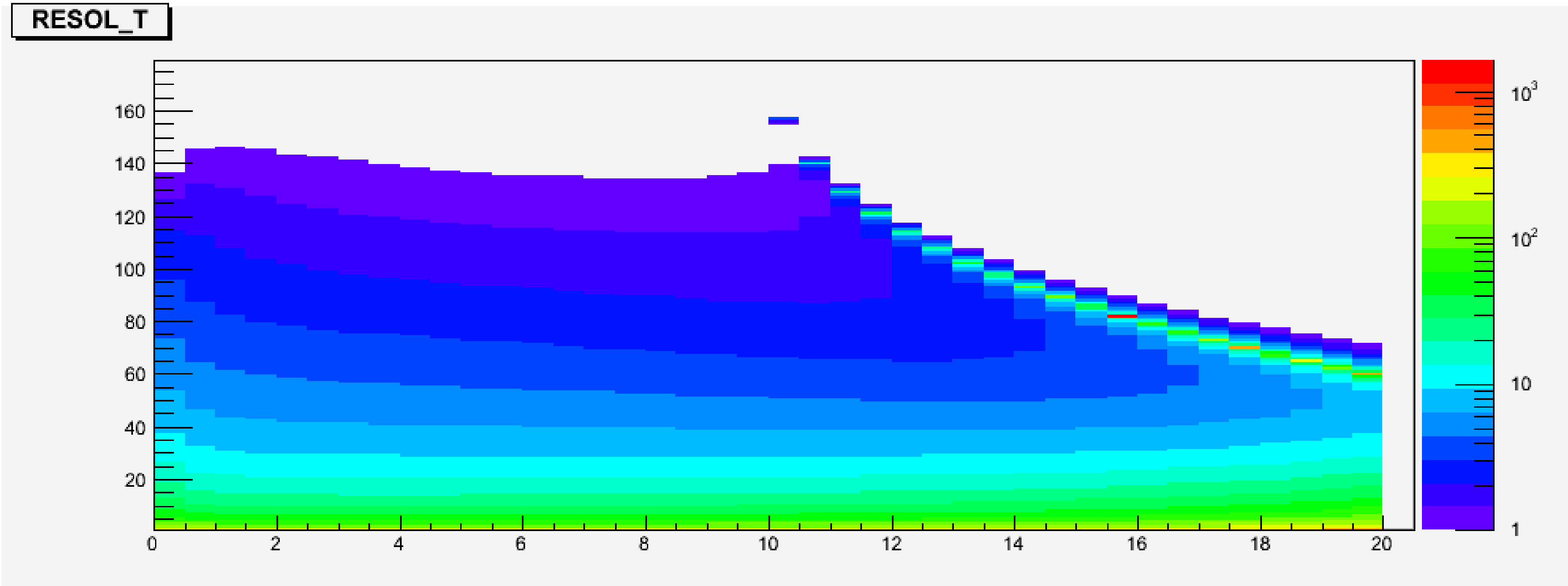
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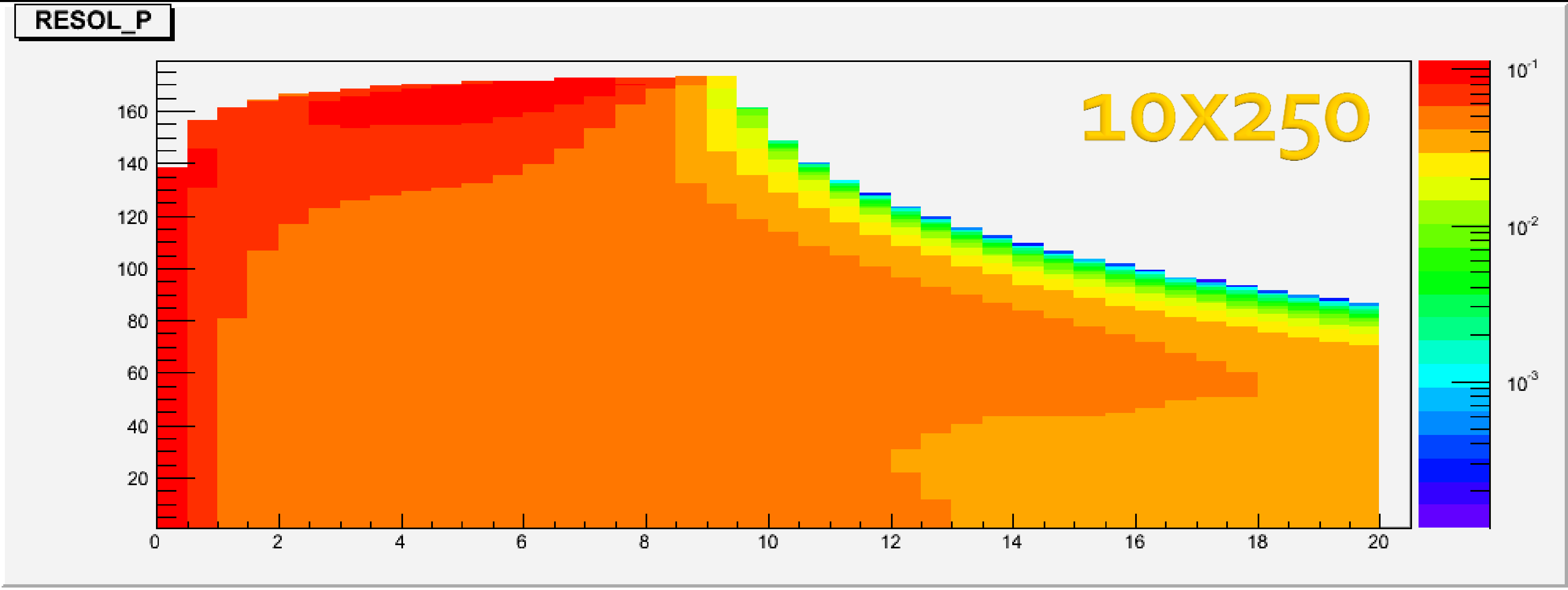
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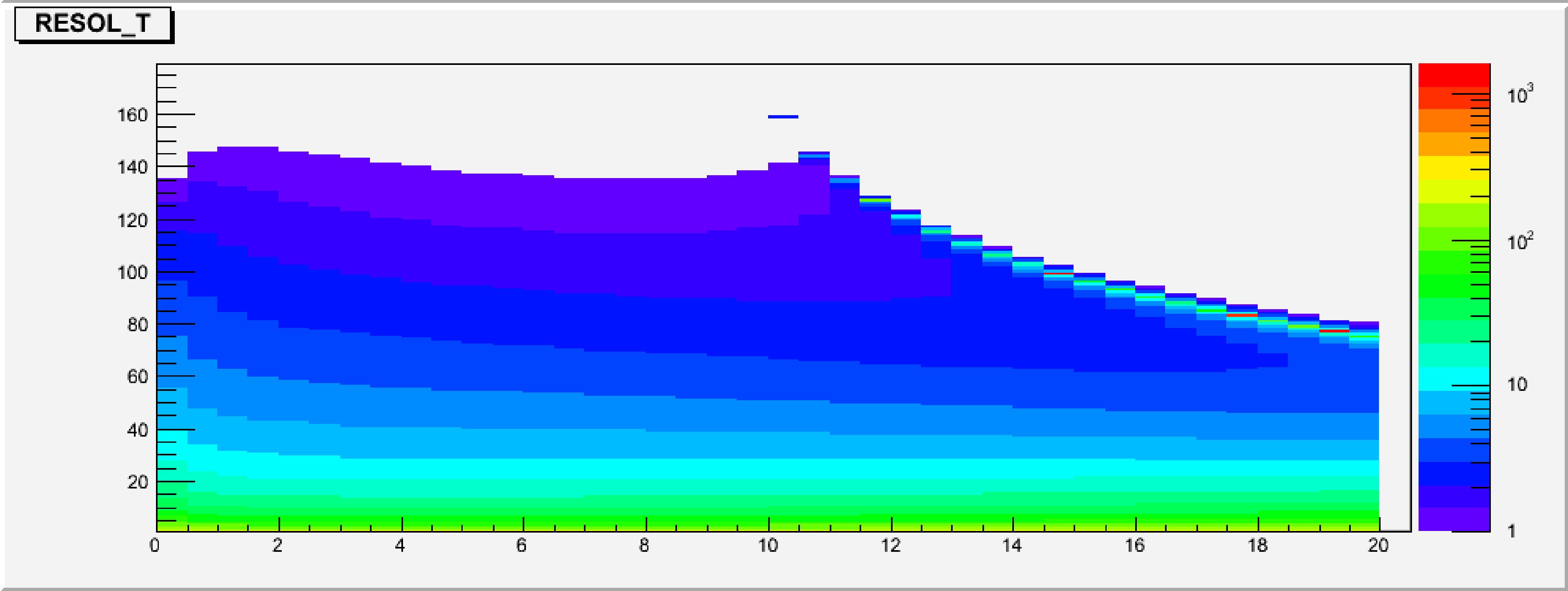
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Next for Chiapas: "F" m…

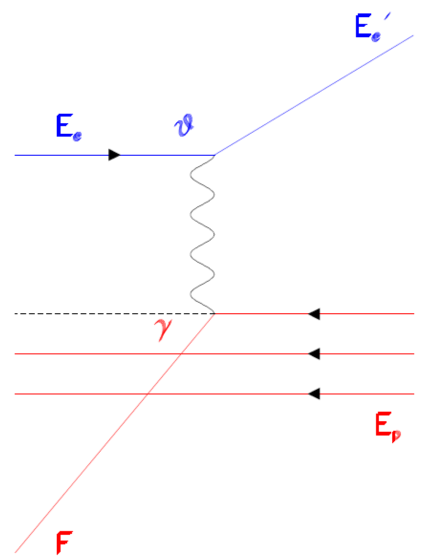
- Reconstruction of event kinematics

- Electron method: scattered electron

$$x_e = \frac{Q_e^2}{s y_e} = \frac{E'_e \cos^2\left(\frac{\theta'_e}{2}\right)}{E_p\left(1 - \frac{E'_e}{E_e}\sin^2\left(\frac{\theta'_e}{2}\right)\right)}$$

$$y_e = 1 - \frac{E'_e}{2E_e}\left(1 - \cos\theta'_e\right) = 1 - \frac{E'_e}{E_e}\sin^2\left(\frac{\theta'_e}{2}\right)$$

$$Q_e^2 = 2E_eE'_e\left(1 + \cos\theta'_e\right) = 4E_eE'_e\cos^2\left(\frac{\theta'_e}{2}\right) = \frac{p_{T,e}^2}{1 - y_e}$$



$$F = \frac{p_{T,h}^2 + (E - p_z)_h^2}{2\,(E - p_z)_h}$$

$$\cot\gamma = \frac{p_{T,h}^2 - (E - p_z)_h^2}{p_{T,h}^2 + (E - p_z)_h^2}$$

- Jacquet-Blondel method: hadronic final state

$$x_{JB} = \frac{Q_{JB}^2}{s y_{JB}}$$

$$y_{JB} = \frac{(E - p_z)_h}{2E_e}$$

$$Q_{JB}^2 = \frac{p_{T,h}^2}{1 - y_{JB}}$$

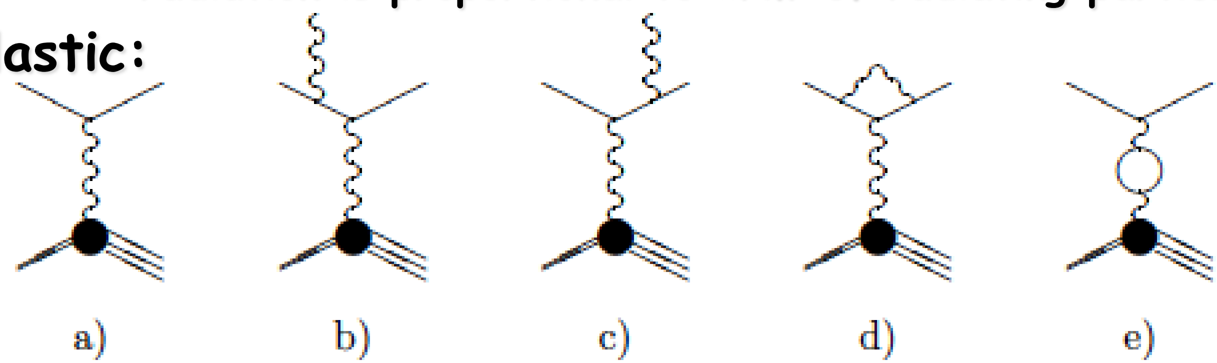
$$p_{T,h}^2 = \left(\sum_h p_{x,h}\right)^2 + \left(\sum_h p_{y,h}\right)^2$$

$$(E - p_z)_h = \sum_h (E_h - p_{z,h})$$

JB Kinematics uses PID hadrons
Fills in extremely low x
(e @ beam pipe)

Some Info on Internal RadCors

- Inclusive cross section
  - $\sigma_{tot} = \sigma_{ela} + \sigma_{qela} + \sigma_{inel} + \sigma_{v}$
    - for all parts photons can be radiated from the incoming and outgoing lepton, high Z-material Compton peak.
      - radiation is proportional to Z^2 of target, for elastic scattering like bremsstrahlung
      - radiation is proportional to $1/m^2$ of radiating particle
  - elastic:



initial final vacuum loops

  - quasi-elastic: scattering on proton in nuclei
    - proton stays intact
    - nuclei breaks up
  - two photon exchange? Interference terms?

Summary

- Chiapas does semi-analytical calculation of detector resolution and coverage necessary to achieve physics goals.
- JB Kinematics are simple extension:
  - Identifies needs for hadron PID.
- Initial (& final) state radiation can be handled as "limiters" in follow-up calculation:
  - Don't make spectrometer better than rad limit.
  - "Fold" instead of "Unfold".